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# Vibration control of suspension systems using a proposed neural network

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**Abstract**

This paper presents a neural scheme for controlling a bus suspension system. The suspension system, designed as quarter bus model, is used to simplify the problem to a one-dimensional spring–damper system. The proposed controller is such that the system is always operating in a closed loop, which should lead to better performance characteristics. For comparison, PID, PI and PD controllers are also utilized to control the bus suspension system. Simulation results give superior performance of the proposed neural control scheme. It was also shown that the designed suspension control system displayed robust performance with system model uncertainties.
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## 1. Introduction

Recently, researchers have been testing a bus axle and suspension system using various dampers, including adjustable units, for optimal damping. The test results could be integrated into the phase one tool, along with the capacity to model coupling between axles, suspension, seat, and driver. The next step would be to validate the enhanced version through on-the-road trials.

A design of a mixed suspension system (an actuator in tandem with a conventional passive suspension) for the axletree of a road vehicle based on a linear model with 4 degrees-of-freedom (d.o.f.) has been realized in Ref. [1]. The authors proposed an optimal control law that was aimed at optimizing the suspension performance while ensuring that the magnitude of the forces generated by the two actuators and the total forces applied between wheel and body never exceeded given bounds.

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A simple and convenient matrix expression has been derived for the performance index in the case of a linear vehicle model with 2 d.o.f. and a preview active suspension, subject to a unit step road input and employing optimal control [2]. The usual quadratic integral-type performance index was assumed and the effect of an additional form of constraint was briefly described. The effects of preview time on the performance index and the optimal feed-forward control were illustrated graphically for a particular example.

A modified discrete time preview control algorithm for active and semi-active suspension systems based on a simple mathematical 4 d.o.f. half-car model has been derived by Youn [3]. The discrete time preview control laws for ride comfort were employed in a simulation. An algorithm for MIMO system contains control strategies reacting against body forces that occur at cornering, accelerating, braking, or under payload, in addition to road disturbances.

Construction of an active suspension system for a quarter car model using fuzzy reasoning has been investigated Ref. [4]. In this research, an active control was utilized as a weighted sum of the *de-fuzzificated* values of the outputs in single input rule modules and generated by using a pneumatic actuator.

A fuzzy controller has been designed for automotive active suspension systems [5]. In this investigation, a half-car model was employed in order to consider the pitch angle of the body and the coupling dynamics of front and rear wheels. It was assumed that the three measurements of body acceleration, front suspension deflection and rear suspension deflection were available. The fuzzy control rules were separately designed for each measurement. After the fuzzy control rules were determined, a genetic algorithm was applied to tune the membership functions of these control rules. The performance of the designed system was evaluated with respect to these disturbance models, and it has been shown that the designed active suspension system provided good performance in improving ride quality and maintaining vehicle manoeuvrability.

A control scheme of an active suspension system using a quarter car model has been proposed by Kim and Ro [6]. The authors have shown that due to the presence of non-linearities such as a hardening spring, a quadratic damping force and the 'tyre lift-off' phenomenon in a real suspension system, it was very difficult to achieve desired performance using linear control techniques. To ensure robustness for a wide range of operating conditions, a sliding mode controller has been designed and compared with an existing non-linear adaptive control scheme in the literature. The sliding mode scheme utilizes a variant of a sky–hook damper system as a reference model which does not require real-time measurement of road input.

An investigation of the variation of vertical vibrations of vehicles using a radial basis neural network (RBNN) has been presented in Refs. [7,8]. The RBNN was employed to predict the desired values of amplitude of acceleration for different road conditions such as concrete, waved stone, block paved and country roads. The proposed neural system was also tested for different natural frequencies and the ratios of damping.

The problem of the design of a non-linear hybrid car suspension system for ride qualities using neural networks (NNs) has been presented [9]. In their investigation, a NN controller was proposed, which corresponded to a Taylor series approximation of the non-linear control function and the NN was due to the numerous local minima trained using a semi-stochastic parameter optimization method. Two cases A and B (continuous and discontinuous operation) were investigated and numerical examples illustrated design methodology.

In this paper, an adaptive control scheme was investigated to control a bus suspension system. The robustness of the proposed scheme was presented through computer simulation and the efficacy of the scheme is shown both in the time and amplitude domains.

The paper is organized in the following manner. Section 2 gives the main structure of the suspension system used. Standard PID and neural network control methods are presented in Section 3. Simulation results and discussion of the control schemes are given in Section 4. The paper is concluded with Section 5.

## 2. Bus suspension system

Designing an automatic suspension system for a bus turns out to be an interesting control problem. When the suspension system is designed, a quarter bus model (one of the four wheels) is used to simplify the problem to a one-dimensional spring–damper system. A schematic diagram of the suspension system is shown in Fig. 1. Where $Y_1$ is the displacement of the bus body mass, $Y_2$ is displacement of the suspension mass, $W$ is the road displacement and $F$ is the forces of the bus body mass and suspension mass. The dynamic parameters of the bus suspension system are given in Table 1.

An acceptable and good bus suspension system should have satisfactory road holding ability, while still providing comfort when riding over bumps and holes in the road. When the bus is experiencing any road disturbance (i.e., potholes, cracks, and uneven pavement), the bus body should not undergo large oscillations and the oscillations should dissipate quickly. Since the distance $(Y_1 - W)$ is very difficult to measure, and the deformation of the tyre $(Y_2 - W)$ is negligible, the distance $(Y_1 - Y_2)$ can be used instead of $(Y_1 - W)$ as the output in this problem. The road disturbance $(W)$ in this problem would be simulated by a step input. This step could represent the bus coming out of a pothole.

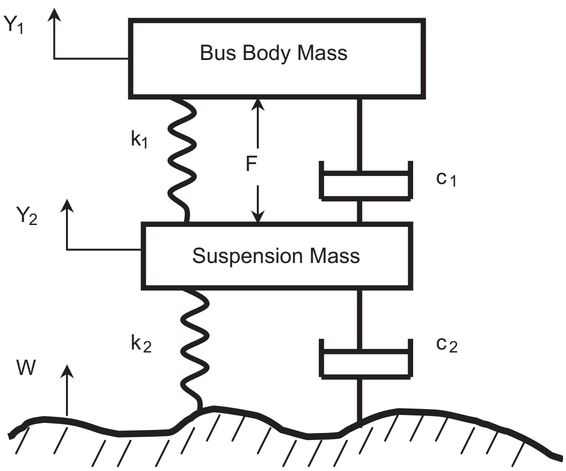


Fig. 1. Schematic representation of the suspension model.

Table 1
Dynamic parameters of the suspension system

| Parameter | Value |
|---|---|
| Bus body mass ($m_1$) | 2500 kg |
| Suspension mass ($m_2$) | 320 kg |
| Spring constant of suspension ($k_1$) | 80 000 N/m |
| Spring constant of wheel and tire ($k_2$) | 500 000 N/m |
| Damping constant of suspension ($c_1$) | 350 Ms/m |
| Damping constant of wheel and tyre ($c_2$) | 15 020 Ms/m |

## 3. Control schemes

For completeness, this section briefly reviews the control schemes, which are PID control, adaptive control and the adaptive control proposed in this paper.

### 3.1. PID control

PID control is perhaps the most widely used control method. It can provide fast response, good system stability and small steady state errors in a linear system with known parameters.

As its name implies, a PID controller consists of three parts: proportional, integral and derivative. Assuming that each amplitude is completely decoupled and controlled independently from other amplitudes, the control input $\mathbf{F}(t)$ is given by

$$\mathbf{F}(t) = \mathbf{K}_P\mathbf{e}(t) + \mathbf{K}_I \int \mathbf{e}(t)\,\mathrm{d}t + \mathbf{K}_D \frac{\mathrm{d}\mathbf{e}(t)}{\mathrm{d}t}. \tag{1}$$

In equation, $\mathbf{e}(t)$ is the control error

$$\mathbf{e}(t) = \mathbf{Y}_d(t) - \mathbf{Y}(t), \tag{2}$$

where $\mathbf{Y}_d(t)$ is the desired car amplitude of displacement and $\mathbf{Y}(t)$ is the current measured car amplitude. $\mathbf{K}_P$ is called the proportional gain, $\mathbf{K}_I$ the integral gain and $\mathbf{K}_D$ the derivative gain, all of which are $(n \times n)$ diagonal matrices where $n = 1$ is the number of amplitudes. PID controllers for articulated buses face two main difficulties, non-linearity and cross-coupling, as can be seen in the dynamic equations in the appendix.

### 3.2. Proposed neural controller

The neural networks employed in this work were of the recurrent type. Recurrent networks have the advantage of being able to model dynamic systems accurately and in a compact form. A recurrent network can be represented in a general diagrammatic form as illustrated in Fig. 2(a). This diagram depicts the hybrid hidden layer as comprising a linear part and a non-linear part and shows that, in addition to the usual feedforward connections, the networks also have feedback connections from the output layer to the hidden layer and self-feedback connections in the hidden layer. The reason for adopting a hybrid linear/non-linear structure for the hidden layer will be evident later.

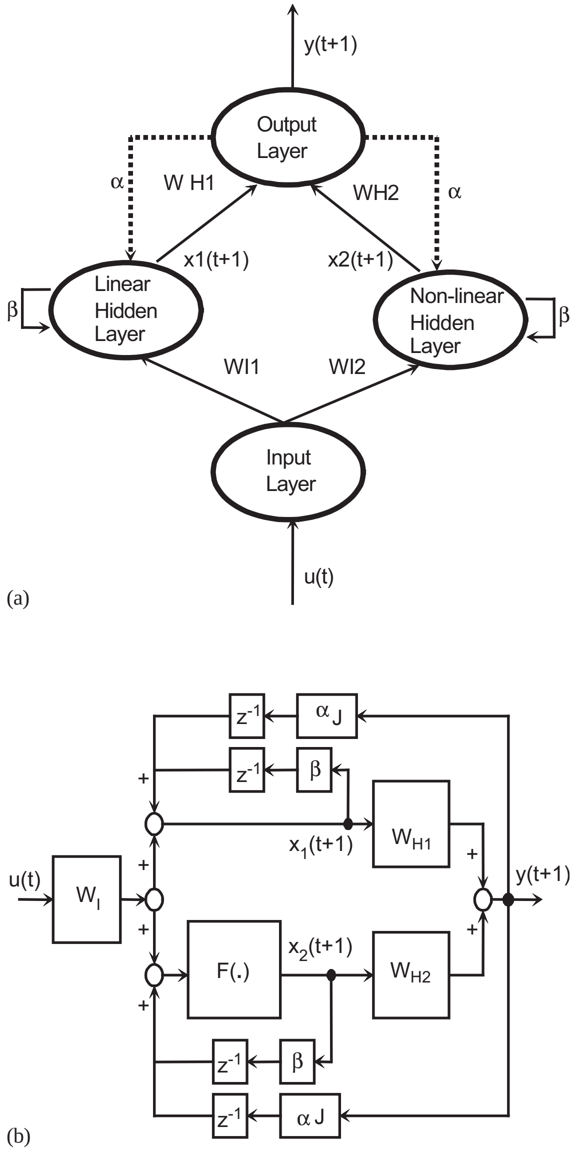


Fig. 2. (a) Recurrent hybrid network structure, (b) Block diagram of recurrent hybrid network.

At a given discrete time $t$, let $\mathbf{u}(t)$ be the input to a recurrent hybrid network, $\mathbf{y}(t)$, the output of the network, $\mathbf{x}_1(t)$ the output of the linear part of the hidden layer and $\mathbf{x}_2(t)$ the output of the non-linear part of the hidden layer.

The operation of the network is summarized by the following equations (also see Fig. 2(b)):

$$\mathbf{x}_1(t+1) = \mathbf{W}^{I1}\mathbf{u}(t+1) + \beta\mathbf{x}_1(t) + \alpha\mathbf{J}_1\mathbf{y}(t), \tag{3}$$

$$\mathbf{x}_2(t+1) = \mathbf{F}\{\mathbf{W}^{I2}\mathbf{u}(t+1) + \beta\mathbf{x}_2(t) + \alpha\mathbf{J}_2\mathbf{y}(t)\}, \tag{4}$$

$$\mathbf{y}(t+1) = \mathbf{W}^{H1}\mathbf{x}_1(t+1) + \mathbf{W}^{H2}\mathbf{x}_2(t+1), \tag{5}$$

where $\mathbf{W}^{I1}$ is the matrix of weights of connections between the input layer and the linear hidden layer, $\mathbf{W}^{I2}$ is the matrix of weights of connections between the input layer and the non-linear hidden layer, $\mathbf{W}^{H1}$ is the matrix of weights of connections between the linear hidden layer and the output layer, $\mathbf{W}^{H2}$ is the matrix of weights of connections between the non-linear hidden layer and the output layer, $\mathbf{F}\{\}$ is the activation function of neurons in the non-linear hidden layer and $\alpha$ and $\beta$ are the weights of the self-feedback and output feedback connections. $\mathbf{J}_1$ and $\mathbf{J}_2$ are respectively $(n_{H1} \times n_O)$ and $(n_{H2} \times n_O)$ matrices with all elements equal to 1, where $n_{H1}$ and $n_{H2}$ are the numbers of linear and non-linear hidden neurons, and $n_O$, the number of output neurons.

If only linear activation is adopted for the hidden neurons, the above equations simplify to

$$\mathbf{y}(t+1) = \mathbf{W}^{H1}\mathbf{x}(t+1), \tag{6}$$

$$\mathbf{x}(t+1) = \mathbf{W}^{I1}\mathbf{u}(t+1) + \beta\mathbf{x}(t) + \alpha\mathbf{J}_1\mathbf{y}(t). \tag{7}$$

Replacing $\mathbf{y}(t)$ by $\mathbf{W}^{H1}\ \mathbf{x}(t)$ in Eq. (7) gives

$$\mathbf{x}(t+1) = (\beta\mathbf{I} + \alpha\mathbf{J}_1\mathbf{W}^{H1})\mathbf{x}(t) + \mathbf{W}^{I1}\mathbf{u}(t+1), \tag{8}$$

where $\mathbf{I}$ is a $n_{H1} \times n_{H1}$ identity matrix.

Eq. (8) is of the form

$$\mathbf{x}(t+1) = \mathbf{A}\ \mathbf{x}(t) + \mathbf{B}\ \mathbf{u}(t+1), \tag{9}$$

where $\mathbf{A} = \beta\mathbf{I} + \alpha\mathbf{J}\mathbf{W}^{H1}$ and $\mathbf{B} = \mathbf{W}^{I1}$. Eq. (9) represents the state equation of a linear system of which $\mathbf{x}$ is the state vector. The elements of $\mathbf{A}$ and $\mathbf{B}$ can be adjusted through training so that any arbitrary linear system of order $n_{H1}$ can be modelled by the given network. When non-linear neurons are adopted, this gives the network the ability to perform non-linear dynamic mapping and thus model non-linear dynamic systems. The existence in the recurrent network of a hidden layer with both linear and non-linear neurons facilitates the modelling of practical non-linear systems comprising linear and non-linear parts.

Fig. 3 shows the proposed control system for a bus suspension. The system comprises a $P$ controller and an NN controller, which is a recurrent hybrid network used to model inverse dynamics of the suspension system. The NN is trained on-line during the control to make the

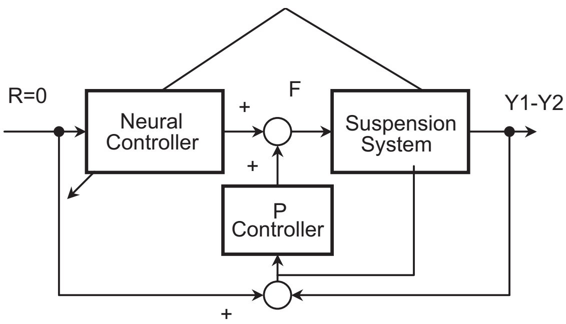


Fig. 3. Block diagram of the proposed neural control system.

system be able to adapt to changes. Relative amplitude ($R = 0$) is used as reference signals to the controller.

## 4. Simulation results

The performance of the proposed adaptive control scheme is illustrated in this section through a series of simulations.

The control architecture illustrated in Fig. 2 was implemented on a Pentium III 500 MHz personal computer using MATLAB software [10]. The dynamic equations of the suspension system are given in the appendix. The simulation results are presented below. They illustrate the effects of different controllers on the performance of the control systems.

First, using the proposed neural control system of Fig. 3, the suspension system was controlled. Structural and learning parameters of the proposed neural network are given in Table 2. From Fig. 4 the overshoot is initially very small. The output has an overshoot less than 1% and settling time shorter than 1 s.

Fig. 5 shows results of the PI controller system. Here, the overshoot is small and the settling time is also short. The output ($Y_1 - Y_2$) has an overshoot less than 2% and a settling time shorter than 1.5 s.

Using the PD controller for suspension system, the results of the overshoot and settling time are shown in Fig. 6. The gain parameters, $K_P$ and $K_D$, of the PD controller was obtained by the Ziegler–Nichols closed-loop method and set to $K_P = 126\,143$, $K_D = 283\,452$ [11]. From the figure, the settling time is not stable and overshoot is very big. The output has an overshoot more than 10% and settling time is long and not stable.

The results are shown Fig. 7 for the case with the PID controller. For comparison, the performance of the neural controller is illustrated in Fig. 3 with the increased gains. Again, the gain parameters of the controllers were chosen empirically and again set to $K_P = 126\,143$, $K_I = 167\,433$ and $K_D = 283\,452$ [11]. As can be derived from the figures, overshoot and settling time are not acceptable for practical applications. The system has the output ($Y_1 - Y_2$) with overshoot more than 5% and a settling time longer than 3 s.

It can be observed from the above simulation results that the controller developed in this paper can guarantee the stability of the adaptive system in the presence of the modelling uncertainties and smaller tracking errors could be achieved with smaller gain parameters of the controller. Also, the proposed neural control scheme results are better than those of standard control schemes.

Table 2
Structural and training parameters of neural controllers

| Controller | $\eta$ | $\mu$ | $\alpha$ | $\beta$ | $n$ | $N$ | $A_F$ |
|---|---|---|---|---|---|---|---|
| NC | 0.0001 | 0.01 | 0.8 | 0.8 | 6+6 | 50 000 | $H_T$ |

Note. $\eta$, learning term; $\mu$, momentum term; $\alpha$, feedback gain from output layer to hidden layer; $\beta$, feedback gain from hidden layer to itself; $n$, linear + non-linear neurons in the hidden layer; $N$, iteration numbers; $A_F$, activation function for non-linear neurons; $H_T$, hyperbolic tangent.

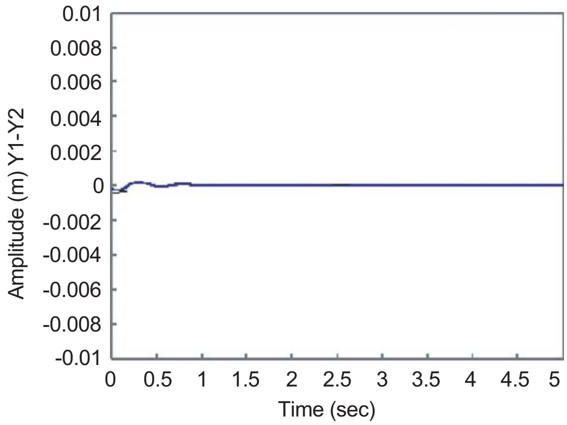


Fig. 4. Amplitude variations of the suspension system using proposed adaptive neural controller.

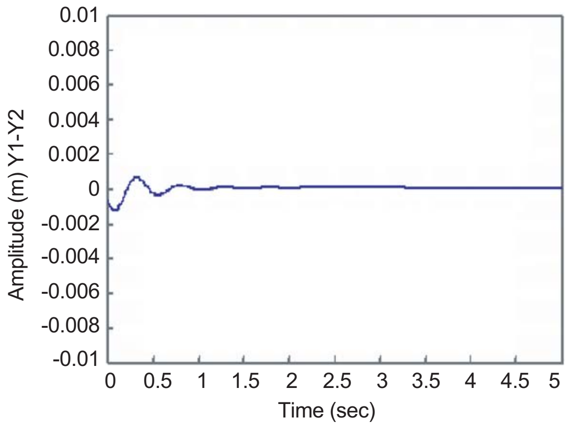


Fig. 5. Amplitude variations of the suspension system using proposed adaptive PI controller.

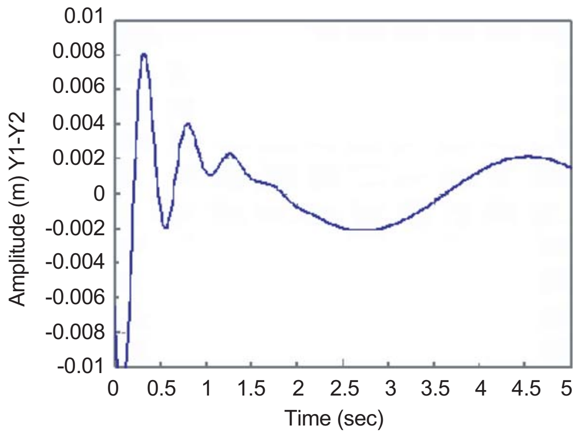


Fig. 6. Amplitude variations of the suspension system using PD controller.

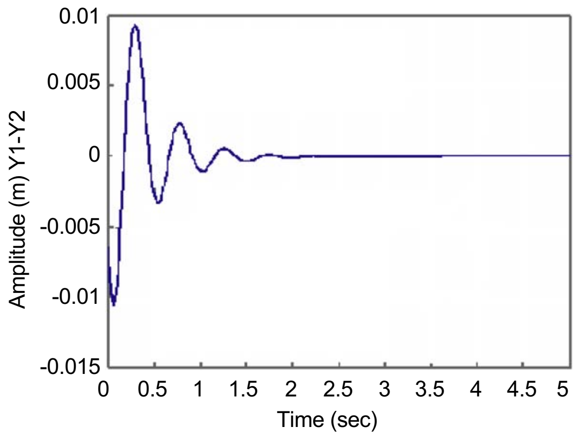


Fig. 7. Amplitude variations of the suspension system using standard PID controller.

## 5. Conclusion

Suspension systems of buses are multivariable dynamic systems for which it is difficult to derive mathematical models. Therefore, analytical control schemes based on such models are complex to construct and generally do not perform well in practice. On the other hand, control schemes incorporating adaptive controllers that can control the unmodelled part of the suspension dynamics are simple to realize and can yield accurate control. This paper has described a proposed neural control scheme for suspensions of the buses. The scheme employs a proposed adaptive tuning as the controller. Because of its inherent ability to represent dynamics, the controller is easy to adapt for control tasks. The simulation results obtained have confirmed the feasibility of the proposed adaptive control scheme and the superior robustness of the decoupled scheme where the suspension was separately controlled over the coupled scheme where only one controller was employed for one suspension.

The Model Reference Adaptive Control approach works well and can give better control than a simple PID scheme when an accurate dynamics model of the system is available. However, in practical simulations, it is very difficult, if not impossible, for the parameters associated with the suspension system model to be determined exactly.

Further work should be considered if a robust adaptive neural controller is to be designed to stabilize the sideways rocking amplitude of the suspension systems. Also, for greater user confidence, it would be necessary to prove the stability characteristics of the proposed neural adaptive control schemes theoretically as well as experimentally.

## Acknowledgements

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## Appendix A. Equations of motion

From Fig. 1 and Newton's law, the dynamic equations of the suspension system can be expressed as follows:

$$m_1 \ddot{Y}_1 = -c_1(\dot{Y}_1 - \dot{Y}_2) - k_1(Y_1 - Y_2) + F,$$

$$m_2 \ddot{Y}_2 = -c_1(\dot{Y}_1 - \dot{Y}_2) + k_1(Y_1 - Y_2) + c_2(\dot{W} - \dot{Y}_2) + k_2(W - Y_2) - F.$$

### *A.2. Transfer function equation of suspension system*

Assume that all of the initial conditions are zero, so these equations represent the situation when the bus wheel goes up a bump. The dynamic equations above can be expressed in a form of transfer functions by taking Laplace transform of the above equations. The derivation from the above equations of the transfer functions $G_1(s)$ and $G_2(s)$ of output, $Y_1 - Y_2$, and two inputs, $F$ and $W$, is as follows:

$$(m_1 s^2 + c_1 s + k_1) Y_1(s) - (c_1 s + k_1) Y_2(s) = F(s),$$

$$-(c_1 s + k_1) Y_1(s) + (m_2 s^2 + (c_1 + c_2)s + (k_1 + k_2)) Y_2(s) = (c_2 s + k_2) W(s) - F(s),$$

$$\begin{bmatrix} (m_1 s^2 + c_1 s + k_1) & -(c_1 s + k_1) \\ -(c_1 s + k_1) & (m_2 s^2 + (c_1 + c_2)s + (k_1 + k_2)) \end{bmatrix} \begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ (c_2 s + k_2) W(s) - F(s) \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} (m_1 s^2 + c_1 s + k_1) & -(c_1 s + k_1) \\ -(c_1 s + k_1) & (m_2 s^2 + (c_1 + c_2)s + (k_1 + k_2)) \end{bmatrix},$$

$$\Delta = \det \begin{bmatrix} (m_1 s^2 + c_1 s + k_1) & -(c_1 s + k_1) \\ -(c_1 s + k_1) & (m_2 s^2 + (c_1 + c_2)s + (k_1 + k_2)) \end{bmatrix}.$$

Find the inverse of matrix $\mathbf{A}$ and then multiple with inputs $F(s)$ and $W(s)$ on the right-hand side as follows:

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} (m_2 s^2 + c_2 s + k_2) & (c_1 c_2 s^2 + (c_1 k_2 + c_2 k_1)s + k_1 k_2 \\ -m_1 s^2 & (m_1 c_2 s^3 + (m_1 k_2 + c_1 c_2)s^2 + (c_1 k_2 + c_2 k_1)s + k_2 k_1) \end{bmatrix} \begin{bmatrix} F(s) \\ W(s) \end{bmatrix}.$$

When the control input $F(s)$ only was considered, $W(s)$ was set to $W(s) = 0$. Thus, the transfer function $G_1(s)$ can be written as follows:

$$G_1(s) = \frac{Y_1(s) - Y_2(s)}{F(s)} = \frac{(m_1 + m_2)s^2 + c_2 s + k_2}{\Delta}.$$

When the disturbance input $W(s)$ was only considered, $F(s)$ was set to $F(s) = 0$. Thus, the transfer function $G_2(s)$ can be written as the following:

$$G_2(s) = \frac{Y_1(s) - Y_2(s)}{W(s)} = \frac{(-m_1 c_2 s^3 - m_1 k_2 s^2)}{\Delta}.$$

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